



Financial Products: exercises

Preliminary version – Work in progress

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Foreword

The following exercises have been selected from various online sources to help you practice the notions introduced in the lecture "Financial products and Introduction to Finance". More exercises will be added in the future.

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Chapter 1

Exercises

Unless otherwise specified, in what follows interest rates are assumed to be continuously compounded and per annum. Hence, the discount factor is calculated for an interest rate equals r for a maturity T shall be computed as e^{-rT} .

1.1 Forward contracts

Exercise 1

A share current exchange rate equals 80 USD. Interest rates are equals 5%, regardless of the maturity. In 6 month a dividend will be paid. Determine the forward price for delivery in 9 months:

- (a) the dividend equals 0 USD
- (b) the dividend equals 3 USD.
- (c) the dividend equals 4% of the value of the stock at that time

Solutions

- (a) 83.06 USD
- (b) 80.02 USD
- (c) 79.86 USD

Exercise 2

The current forward price of a share to be delivered in six months equals 58 USD. Interest rates (continuously compounded) are equals 10% per annum, regardless of the maturity. Determine the spot price if:

- (a) no intermediary dividend is paid

- (b) a dividend of 2 USD is paid in 3 months
- (c) a dividend equals 8% of the value of the stock at that time is paid in 3 month
- (d) a dividend equals 8% of the value of the stock at that time is paid in 6 months

Solutions

- (a) 55.17 USD
- (b) 57.12 USD
- (c) 59.59 USD
- (d) 59.59 USD

Exercise 3

The exchange rate of US dollars is today 8.50 SEK per dollar. The forward price of a dollar to be delivered in six months is 8.40 SEK. If the Swedish six month interest rate is 4%, determine the US six month interest rate.

Solution: 6.37%

Exercise 4

We are August 1st. The forward price of a US dollar to be delivered on December 31st is 0.94630 Euros. The forward price of a US dollar to be delivered on at the end of June next year is 0.95152 Euros. Assuming a flat term structure for both countries and that the Euro interest rate equals 4%, what is the level of US rates?

Solution: 2.90%

Exercise 5

The S&P index spot price is 1100 and the 9-month zero-coupon bond price, in % term, is $ZC = 97.00$. You observe a 9-month forward price of 1109. The dividend is supposed to be paid right before the Forward contract maturity.

- (a) What dividend value is implied by this forward price?
- (b) Suppose you believe the dividend that will be paid will be equals 15. What strategy would you undertake?

Solution 5

- (a) Let D be the value of that dividend. By absence of arbitrage, necessarily, the expected dividend value shall be equals $D = 1134 - 1109 = 35$.
- (b) If the dividend paid is 15 instead of 35 then the real cost of carry will be higher than the one priced in the forward price. Hence you shall go long the forward and short the S&P index and pocket the difference i.e. 20.

Exercise 6

Boeing just signed a contract to sell a 777 aircraft to Air France. Air France will be billed \$300 million which is payable in one year. Air France has the major part of its cash in Euros. The current spot exchange rate is 1.090\$/€ and the one-year forward rate is \$1.112/€. The annual interest rate available to Air France is 5.0% in the U.S. and 3.0% in Europe. Air France is concerned with the volatile exchange rate between the dollar and the euro and would like to hedge its currency exposure.

- (a) Describe qualitatively the risk which Air France is taking
- (b) Air France is considering a first hedging alternative: which would consist of changing forward the amount of USD received thanks to a currency Forward contract. What would be the final euro value of such operation ?
- (c) The first hedging alternative would consist of lending an amount of USD such that the amount due in 1 year would be exactly equals 300M. The amount of USD received could be changed spot in euro and the proceeds used to buy a Zero Coupon. What would be the final euro value of such operation ?

Solution 6

- (a) Since Air France will have to pay USD in one year time, its risk is that USD appreciates against EUR.
- (b) Converting the proceeds using a forward contract of the sale would yield in euro $V_1 = \$300,000,000/1.112 = \text{€}269,784,173$.
- (c)
 - Borrowing USD: $300,000,000/e^{0.05} = \$285,368,827$
 - Changing spot in EUR: $\$285,368,827/1.09 = \text{€}261,806,264$
 - Investing in ZC: $\text{€}261,806,264 \times e^{0.06} = \text{€}269,779,451$.

1.2 Options

Exercise 7

(Use a B&S calculator available on the Web) Determine the price of a European futures call option on a barrel of crude oil to be delivered in 4 months. This is also the time of maturity of the option. The future price today is $F_0 = \$25.00$, the strike price is $\$23.00$ and the volatility of the futures price is estimated to be 25% for one year. The risk-free interest rate is 9% per year.

Solution 7

Use a B&S calculator available on the Web. The appropriate formula is the Black formula for futures. The Black Formula can be obtained using a Black and Scholes formula with the following parameters:

- Spot = Forward = 25
- Strike, Maturity, Rate at their level
- Div = Rate to ensure that the spot price equals the forward price

The solution is 2.53.

Exercise 8

Use a B&S calculator available on the Web;

- Calculate the price of a European call option on a share which does not pay dividends before the maturity of the option. The spot share price is 45 SEK, the option matures in 6 months, the volatility is 25% per annum, the option's strike price is 43 SEK and the risk-free interest rate is 5% per annum.
- The same question as above, but now we assume the share pays a dividend of 1 SEK in 3 months, all other assumptions are the same.

Solution :

- 4.832
- 4.176

Exercise 9

Denote X_t the USD vs EUR exchange rate at date t i.e. the value of 1 USD in EUR: $1 \text{ USD} = X_t \text{ EUR}$. We denote $r_{\text{€}}$ and $r_{\text{§}}$ respectively the EUR and the USD continuously compounded rates. In what follows, $F(t, T)$ will denote the forward value of X_t at date $T \geq t$. EUR is considered the domestic currency in which all prices are denominated.

- (a) Give the definition of the forward price $F(t, T)$. At which date is it known ? At which date is it paid ?
- (b) Show that $F(t, T) = X_t e^{(r_{\text{€}} - r_{\text{§}})(T-t)}$
We now wish to price a call with strike K and maturity τ on X_τ . Instead of recalculating everything, we would like to adapt the Black and Scholes formula to obtain such price.
- (c) Use the previous formula and find the price of a call option on the exchange rate as a function of $r_{\text{€}}$, $r_{\text{§}}$, X_0 , σ and τ .
- (d) Compare the intrinsic value of the call and the amount of money you could get in exercising it. Discuss when it makes sense to exercise or not.
- (e) Same question for the put

Solution 9

- (a) The Forward price is a quantity known today (t), paid at capital T in exchange of a random quantity, with no initial exchange of money.
- (b) cash and carry arbitrage, see lecture notes
- (c) Apply the B&S formula with $t = 0$, $r = r_{\text{€}}$ and the value of the forward price:

$$C(T, F_T) = e^{-r_{\text{€}}T} F_T [N(d_1) - KN(d_2)]$$

where

$$\begin{aligned} F_T &= X_t e^{(r_{\text{€}} - r_{\text{§}})T} \\ d_1 &= \frac{-\ln(F_T/K) + \frac{\sigma^2 T}{2}}{\sigma \sqrt{T}} \\ d_2 &= \frac{-\ln(F_T/K) - \frac{\sigma^2 T}{2}}{\sigma \sqrt{T}} \end{aligned}$$

Hence, the foreign rates plays the role of a dividend.

(d) The value of the call is always above the intrinsic value (convexity):

$$C(T, F_T) > (X_t e^{-r_s T} - K e^{-r_{\text{€}} T})^+$$

The value when exercising the American call equals

$$C_{ex} = X_t - K$$

In the option on stocks case, when there is no dividend, necessarily $C(T, F_T) > C_{ex}$ so it is always better to sell the call than to exercise it. In the FX case, if both interest rates are positive, it might be better to exercise the call than to sell it.

Exercise 10

You are given the following:

- The current price to buy one share of XYZ stock is 500,
- The stock does not pay dividends;
- Interest rates are null,
- A European call option on one share of XYZ stock with a strike price of K that expires in one year costs 67,
- A European put option on one share of XYZ stock with a strike price of K that expires in one year costs 19.

Find K .

Solution 10

Because interest rates are equals 0, the price of the ZC with a notional equals K equals K . Now because of the call-put parity, we have

$$67 - 19 = 500 - K$$

hence $K = 452$.

Exercise 11

The PS index has the following characteristics:

- One share of the PS index currently sells for 1,000.
- The PS index does not pay dividends.

Sam wants to secure in the ability to buy this index in one year for a price of 1,025 (even if the price gets eventually lower than 1025). He can do so by buying or selling European put and call options with a strike price of 1,025. The annual effective risk-free interest rate is 5% (linearly compounded). Determine which of the following gives the hedging strategy that will achieve Sam's objective and calculate the cost today of establishing this position.

1. Buy the put and sell the call, receive 2,375
2. Buy the call and sell the put at no cost
3. Buy the call and sell the put, receive 26.25
4. Buy the call and sell the put, pay 26.25

Instead of 1025, which strike would allow to secure the forward buy at zero price ?

Solution 11

A long position of a call option and a short position in a put option with the same strike price provides an exposure on one unit of stock and an equivalent borrowing position proportional to the stock. The difference should be equals

$$Call - Put = 1000 - 1025 \times (1 - 0.05) \sim 26,25$$

The forward price would allow to secure the forward buy at zero price. This is reflected in the call-put equation :

$$1000 - K(1 - 0.05) \Leftrightarrow K \sim 1052,63$$

Exercise 12

Near market closing time on a given day, you lose access to stock prices, but some European call and put prices for a stock are available as follows:

Strike Price	Call Price	Put Price
40	11	3
50	6	8
55	3	11

Explain why this pricing table would provide some arbitrage. Propose an arbitrage strategy. (Hint: use the call/put parity)

Solution 12

The fact that we are near the maturity means that we can make the approximation $Ke^{-rT} \sim K$. We can thus complete the following table and compute which stock price would allow the call-put parity to hold

Strike Price	Call Price	Put Price	Call-Put +Strike
40	11	3	48
50	6	8	48
55	3	11	47

Since the spot price cannot be equals 48 and 47, it means that this option prices table is inconsistent and can be arbitrated. Identifying an arbitrage strategy consists of (trying to) buy what is cheap and selling what is expensive. The call put parity means that you can have a position equals the underlying by entering a (long Call, short Put, Short ZC) strategy.

If you do that with strike 55, this will cost you 47 whereas if you do that with strike 50 (or 40) it will cost you 48.

Now if you enter the (long Call, short Put, Short ZC) strategy for strike 55 and short the (long Call, short Put, Short ZC) strategy for strike 40, you will receive 1 and have no other cash-flows: that is an arbitrage.

Exercise 13

Let X_t be the value of a foreign currency expressed in units of local currency. Let r_f be the -constant- interest rate of the foreign currency and r_l the local interest rate. Let $T \geq t$ a maturity.

- (a) Compute the forward value of the X_t .
 - (a) Does it increase or decrease with r_f ? With r_l ?
 - (b) What is the “cost of carry” of such “asset” ?
 - (c) What would be the risk neutral drift of that exchange rate ?
- (b) Write the equivalent of the Black and Scholes formula in such situation
(solution: https://en.wikipedia.org/wiki/Foreign_exchange_option)

Solution 13

The cash and carry arbitrage consists of borrowing in local currency, change spot, carrying in foreign currency (hence receiving the foreign short rate) and changing again at maturity. As a result:

$$F(t, T) = X_t e^{(r_l - r_f)(T-t)}$$

The forward price equals the expectation of the price at T : $E(X_T|X_t) = F(t, T)$. If X_t is modelled as $dX_t = X_t(\mu dt + \sigma dW_t)$ then X_t can be written as

$$X_T = X_t e^{(\mu - \frac{\sigma^2}{2})(T-t) + \sigma(W_T - W_t)}$$

and hence

$$E(X_T|X_t) = X_t e^{\mu(T-t)}$$

So, under the Risk Neutral Probability $\mu = r_l - r_f$ necessarily holds.

Exercise 14

- (a) In a world without dividends, prove the "Call-Put" Parity i.e.

$$KD(0, T) + Call(S_0, K, T, r) = S + Put(S_0, K, T, r)$$

where $D(0, T) = e^{-rT}$ is the discount factor and S_0 is the initial value of the price and $Call(S_0, K, T, r)$ resp. $put(...)$ the price of the call of strike K and maturity T (resp. put...) obtained with the B&S formula with interest rate r .

- (b) Let F the forward price of S . More precisely $F(t, T) = S_t e^{r(T-t)}$. Thanks to Ito's lemma, compute the drift of $F(t, T)$ under the risk neutral probability. Note $F = F(0, T)$

- (c) Prove that, equivalently

$$K + Call(F, K, T, 0) = F + Put(F, K, T, 0)$$

where the options are now options on the future contract.

Solution 14

Let $\tilde{S}_T$ the (random) price of a given stock at date T , S_0 its price today, and K a given constant amount of money.

Question 1: Mathematically

$$\forall \tilde{S}_T : \tilde{S}_T + (K - \tilde{S}_T)^+ = K + (\tilde{S}_T - K)^+ \quad (1.1)$$

since

- when $\tilde{S}_T \leq K$: $\tilde{S}_T + (K - \tilde{S}_T)^+ = K + (\tilde{S}_T - K)^+ = K$ and
- when $\tilde{S}_T \geq K$: $\tilde{S}_T + (K - \tilde{S}_T)^+ = K + (\tilde{S}_T - K)^+ = \tilde{S}_T$.

If there are no intermediary flows such as coupons or dividends, by no-arbitrage (sometimes referred to as “**the law of one price**”), the prices of both parts of equation 1.1 shall be equal:

$$\text{Price} \left(\tilde{S}_T + \left(K - \tilde{S}_T \right)^+ \right) = \text{Price} \left(K + \left(\tilde{S}_T - K \right)^+ \right)$$

Since the price is linear, necessarily:

$$\underbrace{\text{Price} \left(\tilde{S}_T \right)}_1 + \underbrace{\text{Price} \left(K - \tilde{S}_T \right)^+}_2 = \underbrace{\text{Price} (K)}_3 + \underbrace{\text{Price} \left(\left(\tilde{S}_T - K \right)^+ \right)}_4$$

This proves the statement since

1. is the price of the underlying, S_0 ,
2. the price of the put $\text{Put}(S_0, K, T, r)$,
3. the price of a zero-coupon with notional K and
4. the price of the call, $\text{Call}(S_0, K, T, r)$.

Question 2: We put ourselves in a Black and Scholes model where

$$\begin{aligned} dS_t &= S_t (r dt + \sigma dW_t) \\ S_0 &= S_0 \end{aligned}$$

By definition, the forward price is obtained as

$$F_t = S_t e^{r(T-t)}$$

Ito’s lemma, in this case, yields:

$$\begin{aligned} dF_t &= e^{r(T-t)} dS_t + S_t d \left(e^{r(T-t)} \right) \\ &= F_t (r dt + \sigma dW_t) - F_t r dt \\ &= F_t \sigma dW_t \end{aligned}$$

Which shows that the forward price has a null drift (is a martingale) under the risk neutral probability.

Question 3: This question is very similar to the first one but instead of taking the price of both sides of equation 1.1 we just take the risk neutral expectation:

$$\underbrace{\mathbb{E} \left(\tilde{S}_T \right)}_1 + \underbrace{\mathbb{E} \left(K - \tilde{S}_T \right)^+}_2 = \underbrace{\mathbb{E} (K)}_3 + \underbrace{\mathbb{E} \left(\left(\tilde{S}_T - K \right)^+ \right)}_4$$

Let us look at the different terms.

1) is the expected value of the spot at date T i.e. its forward value $F = S_0 e^{rT}$.

Let us relate 2) to a B&S formula with modified parameters:

$$\begin{aligned}\mathbb{E} \left(K - \tilde{S}_T \right)^+ &= e^{rT} Put(S_0, K, T, r) \\ &= KN(-d_2) - FN(-d_1)\end{aligned}\tag{1.2}$$

Now

$$\begin{aligned}d_1 &= \frac{\ln(S/K) + (r + \sigma^2/2)T}{\sigma\sqrt{T}} \\ &= \frac{\ln(Se^{rT}/K) + (\sigma^2/2)T}{\sigma\sqrt{T}} \\ &= \frac{\ln(F/K) + (\sigma^2/2)T}{\sigma\sqrt{T}}\end{aligned}\tag{1.3}$$

The combination of expressions 1.2 and 1.3 show that $\mathbb{E} \left(K - \tilde{S}_T \right)^+$ is indeed the value of a put option on a future contract in a world with no interest rates hence $Put(F, K, T, 0)$. The same reasoning applies to the call. Eventually, obviously: $\mathbb{E}(K) = K$ which proves the statement.

Exercise 15

Joe believes that the volatility of a stock is higher than indicated by market prices for options on that stock. He wants to speculate on that belief by buying or selling at-the-money options. Determine which of the following strategies would achieve Joe's goal. (see forwardoptionpayoffs.pdf)

1. Buy a strangle
2. Buy a straddle
3. Sell a straddle
4. Buy a butterfly spread
5. Sell a butterfly spread

Solution 15

Best strategy 3. Second best 1. Butterfly spread to unstable.

Chapter 2

Problems

2.1 The Merton model of the firm

Exercise 16

This model is aimed at providing the value of the debt of a company which takes into account its bankruptcy risk. The company structure is modelled as follows. Its business value at time t is denoted S_t . The company initial business value is denoted S_0 . The volatility of the business is denoted σ . This company has been financed by two means:

1. Some initial capital, E_0 , from shareholders ;
2. Some initial debt under the form of a ZC bond of notional D maturity T , from creditors.

The company will not pay any intermediary dividend. The riskless interest rate is denoted r . The interest rate paid by the company equals $r+s$ where s is a spread above the riskless rate which reflects the fact that lending money to the company can be risky. This means that the value of the risky ZC today equals $De^{-(r+s)T}$. In that framework, the bankruptcy event is defined as $S_T < D$.

At maturity, shareholders and creditors will share the business value S_T as follows:

1. Creditors are paid first. If the company is not bankrupted, they are redeemed their notional D . If the company is bankrupt, they take all the company value.
2. Shareholders are paid second: if the company is not bankrupted, they get paid $S_T - D$, if it is bankrupted they receive 0.

Let F_T be the T-forward value of the business. The Debt-to-Asset ratio, defined as $l = \frac{D}{F_T}$, is used as a measure of leverage. When $l = 0$, the company is fully unlevered, when it equals 1 is infinitely leveraged.

- (a) Write the final wealth of shareholders as an option payoff. Give the name of this option. Using the Black and Scholes equation, express the value of equity as a function of interest rates r , leverage l , volatility σ and maturity T .
- (b) Using the fundamental law of accounting, express the value of the risky debt. Then, using the call call-put parity, find another expression of the spread.
- (c) Compute the value of the spread s and of the equity for the following sets of parameters and comment:

σ	l	T	r
10%	2	1	0
10%	4	1	0
20%	2	1	0

Table 2.1: parameters

- (d) More generally, who has interest that the company takes more risk ($\sigma \nearrow$)
- (e) We assume $T = 1$, $l = 4$ and $\sigma = 15\%$, $r = 0\%$. At date 0, the company announces that it will pay to shareholders an intermediary dividend equals 5% of the initial value S_0 . What is the new forward value of the company. What is the new leverage ? Compute the new value of the share price and of the spread. Comment.

Solution 16

Q1: Shareholders are long a call option on the firm value at date T , with strike the debt amount to be repaid:

$$(S_T - D)^+$$

Hence, using B&S equation one finds that the financial value of the equity equals

$$C(T, F_T) = e^{-rT} F_T [N(d_1) - lN(d_2)]$$

where

$$F_T = S_0 e^{(r-d)T}$$

$$d_1 = \frac{-\ln(l) + \frac{\sigma^2 T}{2}}{\sigma \sqrt{T}}$$

$$d_2 = \frac{-\ln(l) - \frac{\sigma^2 T}{2}}{\sigma \sqrt{T}}$$

Q2: Since assets are equals liabilities , the forward payoff of creditors equals .

$$D^r = S_T - (S_T - D)^+$$

Then because of call put parity:

$$D^r = De^{-rT} - P(T, F_T)$$

where

$$P(T, F_T) = e^{-rT} F_T [lN(-d_2) - N(-d_1)]$$

Hence

$$\begin{aligned} De^{-(r+s)T} &= e^{-rT} F_T [l - lN(-d_2) + N(-d_1)] \\ \iff e^{-sT} &= \left[N(d_2) + \frac{1}{l} N(-d_1) \right] \end{aligned}$$

$$\boxed{s = -\frac{1}{T} \ln \left(N(d_2) + \frac{1}{l} N(-d_1) \right)}$$

When leverage goes to zero:

$$\begin{aligned} l \rightarrow 0 : -\ln(l) &\rightarrow +\infty, N(d_2) \rightarrow 1, \frac{1}{l} N(-d_1) \rightarrow 0 \\ \Rightarrow s &\rightarrow 0 \end{aligned}$$

When leverage goes to 1:

$$\begin{aligned} l \rightarrow 1 : -\ln(l) &\rightarrow 0, N(d_2) - \frac{1}{l} N(-d_1) \rightarrow 0 \\ \Rightarrow s &\rightarrow +\infty \end{aligned}$$

Q3: When σ increases, the value of the call increases hence the value of equity increases. The value of the put increases as well hence the value of the risky debt decreases. When there is an intermediary dividend, the value of the forward decreases hence the value of the put increases hence the spread increases.

2.2 Structured Products

Exercise 17

We want to create a portfolio which final value would be at least equals its initial value. Let V_0 the initial value of that portfolio. To do so, we aim at buying a ZC bond and some call option on the S&P500. Let r the riskless interest rate, T the considered maturity, σ the volatility of the S&P 500 and d its dividend rate.

- (a) What what notional amount of ZC should be bought to ensure principal protection ? Let D_0 be the initial value of the amount of ZC needed. Express D_0 as a function of V_0 , r and T . Define the “cushion” as $C_0 \equiv V_0 - D_0$. Compute C_0 when $r = 4\%$ and $T = 4$.
- (b) We consider buying call options on the S&P 500. Let S_0 be the price of the S&P today and K the selected strike price. We want to obtain the following payoff:
- if $S_T > S_0 \implies kV_0 \left(\frac{S_T}{S_0} - 1 \right)$
 - if $S_T < S_0 \implies 0$
- (c) Write the payoff. How many calls of which characteristics to you need to obtain this payoff ? Determine k such that the price of those calls equals C_0 .
- (d) Find k when $r = 4\%$ and $T = 4$, $\sigma = 15\%$ and $d = 1.88\%$. Describe the payoff qualitatively. Compare with buying directly the S&P. Why is this not an arbitrage.

Solution 17

(a)

$$C_0 = V_0 (1 - e^{-rT})$$

In the example, with $V_0 = 1$, $C_0 = 14,79\%$

(b) if one buys n calls at the money the payoff is

$$n (S_T - S_0)^+$$

so with $n = \frac{kV_0}{S_0}$ we obtain

$$kV_0 \left(\frac{S_T}{S_0} - 1 \right)^+$$

which is the required payoff. k is then obtained by the following budget equality:

$$C_0 = kV_0 \text{Call}(S_0 = 1, K = 1, \sigma = \sigma, T = T)$$

$$k = \frac{(1 - e^{-rT})}{\text{Call}(S_0 = 1, K = 1, \sigma = \sigma, T = T)}$$

- (c) The price of the call equals $14,8\%$ hence $k = 100\%$. This product provides the performance of the S&P500 when it is positive and zero (i.e. giving the capital back) when it is negative. It appears as an arbitrage but it is not. The structured product has not paid any intermediary dividends while a direct investment in the S&P 500 would have.

2.3 Hedging a Corporate FX Exposure

Exercise 18

Background: A multinational corporation (MNC) based in France expects to receive USD 100 million from a U.S. client in 3 months. The CFO is concerned about exchange rate fluctuations, especially after recent unexpected macroeconomic news that led to increased volatility in the USD/EUR market. By USD/EUR it is meant the value of 1 USD in EUR, denoted X . Let $X_0 = 0.9$ the value as of today. The future random value of the USD/EUR rate is denoted $\tilde{X}_3$.

According to the CEO, the USD should appreciate against the EUR over the next 3 months due to expectations of tighter monetary policy by the FED. Oppositely, the CFO thinks there will be less demand for USD as a safe-haven currency and hence USD should depreciate. They are not very sure about what to do...

The US rates equals $r_{USD} = 5\%$ and the euro rate equals $r_{EUR} = 1\%$. The annual volatility of the USD/EUR equals $\sigma = 10\%$. Rates are continuously compounded.

- (a) Does X increase or decrease when the USD appreciates ? If the CEO is right, should you hedge the risk or not ? Same question if the CFO is right ?
- (b) What is the forward value of the USD/EUR rate ? Write the formula and calculate a numerical approximation. In EUR, what difference would it make to change USD 100 million at the spot rate or at the forward rate ? If the future value of the EUR/USD equals the forward rate, will it mean that the EUR/USD has appreciated or depreciated ?
- (c) Explain why the volatility of $\tilde{X}_3$ should be equals 5%. What is the typical fluctuation of the EUR equivalent of USD 100 million in three months?
- (d) If you don't want to take any risk, what should you do ? Will you obtain the support of the CEO or of the CFO ?
- (e) MNC hesitates to buy a put option or a call option on the USD/EUR. Which one would be appropriate ? Explain why.
- (f) Eventually, the company purchases a European put option on USD 50 million with a strike price of 0.9 USD/EUR, expiring in 3 months. The price of that option equals 1.5% of the notional amount. Graph the following values as a function of X_3 :

- the EUR value of the USD 100 million in three months,

- the EUR value of the put portfolio, including its premia,
 - the sum of both.
- (g) Compare this strategy with a strategy of full hedging (change USD forward). Comment. What would be your own view on the matter ?

Solution 18

- (a) If USD appreciates then one can get more euro with 1 USD and hence X increases. If the CEO is right, then X should increase and MNC should not hedge. If the CFO is right, the opposite is true.
- (b) Since the USD is more rewarding than the euro then the forward rate should be lower than the spot hence the formula of the forward rate is

$$\begin{aligned}
 F_{3m} &= X_0 e^{(r_{EUR} - r_{USD})T} \\
 &= 0.9 \times e^{-4\%/4} \\
 &\approx 0.9 \times (1 - 1\%) \\
 &\approx 0.9 - 0.009 \\
 &\approx 0.891
 \end{aligned}$$

In EUR this means 89.100.100 EUR instead of 90.000.000 EUR hence a difference of approx. 900.000 EUR. If the future value of the spot equals the forward then the USD would have depreciated a bit.

- (c) The variance of a process with i.i.d increments is of the form $\sigma^2 T$ hence standard deviation (volatility) scales as $\sigma\sqrt{T}$. Hence a quarterly standard deviation equals half of the yearly volatility. So the typical fluctuation in 3 months is $89100100 \times 1.05 \text{ EUR} - 89100100 \times 0.95 \text{ EUR} \approx 94M \text{ EUR} - 84M \text{ EUR}$.
- (d) No risk means a fixed quantity of EUR known today hence sell 100M USD forward immediately. The CFO would approve but the CEO would think that MNC is losing the opportunity to benefit from a potential increase of USD against EUR.
- (e) The risk of the company is a decrease of X hence a put option is suitable.
- (f) See Figure 2.1.
- (g) With Full hedging, we would have the yellow curve. Imho, we do not have specific prediction capabilities and the risk should be hedged.

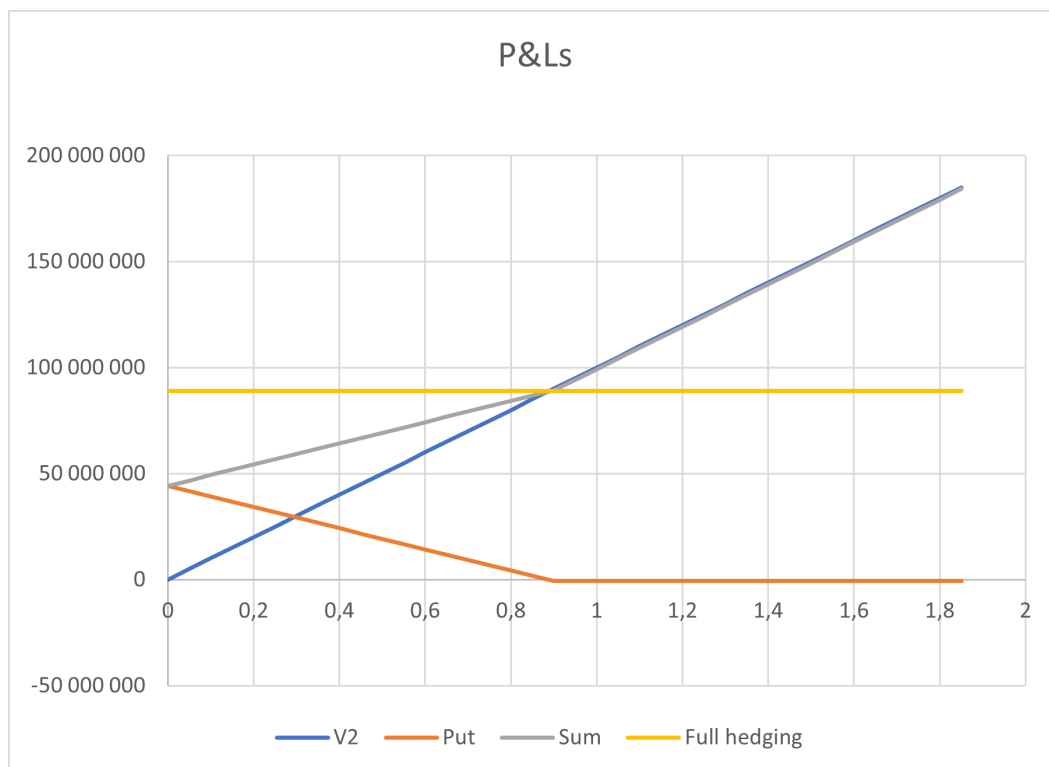


Figure 2.1: Solution FX

T	$r^{eur}(T)$
1	3%
2	3.3%
3	3.6%
4	3.7%

Table 2.2: ZC Yield Curve

2.4 Multi-period / multi-currency financing

Exercise 19

In the following, $r^{eur}(T) = r^{eur}(0, T)$ stands for the continuously compounded euro zero-coupon rate between today and date T . The corresponding discounting factor is obtained by the relationship $P^{eur}(0, T) = e^{-r^{eur}(T)T}$. The Yield curve is displayed in table 2.2. The unknown future ZC rate which will prevail at date for date T is denoted $r^{eur}(t, T)$ and the (known) forward rate between date t and date T is denoted $f^{eur}(0, t, T) = f^{eur}(t, T)$.

A friend of yours needs to borrow 100 000 € for the coming 4 years. He contemplates three possible strategies:

- Strategy SA: contract right now a 4 year ZC loan;
- Strategy SB: borrow money now for the first two years and then, in 2 years time, re-enter into a 2 year loan;
- Strategy SC: borrow money now for the first three years and, in 3 years time, enter into a 1 year loan.

He asks you for some advice.

- Compute the NPV of each strategy as a function of the different rates.
- What is the level of the future ZC rate which makes strategy SB winning or losing as compared to SA ? Comment.
- Same question for SC.
- Eventually, the yield curve remained unchanged during the period. Which strategy has eventually been the most interesting ? Discuss.
- Propose a Python or an R code which could fill the table 2.3 forward rates matrix based on figures in table 2.2:

Somebody brings your attention on the fact that the Japanese yield curve is flat and equals 0%. It also seems that the Yen keeps on depreciating against Euro (see Figure 2.2). So if the Euro value of one Yen is lower tomorrow than it is today, borrowing money in this

	$T - t = 1$	$T - t = 2$	$T - t = 3$	$T - t = 4$
$t = 0$				
$t = 1$				NA
$t = 2$			NA	NA
$t = 3$		NA	NA	NA

Table 2.3: Forward rate matrix



Figure 2.2: EUR/JPY evolution

currency seems to make sense (the debt will lose value). Let $Z(t)$ be the current exchange rate of 1 JPY in EUR and $Z(0) = 0.006$ EUR/JPY.

- (f) Describe a mechanism whereby you would borrow in JPY, change your money in EUR and redeem the money in JPY thanks to changing the money in 4 year time from EUR to JPY. What would be the NPV of this scheme as a function of $Z(4)$?
- (g) For which value of $Z(4)$ is the NPV of that deal positive ? What is the NPV of the gain if $Z(4) = Z(0)$? Comment.

Solution 19

Questions (a) to (c). The three strategies can be parametrized by t , the maturity of the first loan. At t , instead of refunding your loan, you re-borrow money until date T . If N is the notional amount, then the amount to be paid at date t would be $\frac{N}{P(0,t)}$ (which is known). At date t , this amount has to be re-borrowed until date T when $\frac{N}{P(0,T)} \frac{1}{P(t,T)}$ will have to be repaid. As a result the NPV of those strategies can be computed as

$$NPV(t) = N \left(1 - \frac{P(0,T)}{P(0,t)P(t,T)} \right)$$

Note that $P(t, T)$ is random since $r(t, T)$ is not known today. This can also be written as a function of interest rates

$$NPV(t) = N \left(1 - e^{-r(0, T)T + r(0, t)t + r(t, T)(T-t)} \right)$$

The NPV is positive if and only if $-r(0, T)T + r(0, t)t + r(t, T)(T-t) < 0$ which writes

$$r(t, T) \leq f(t, T)$$

where

The previous equation means that the NPV is positive only if the future rates at date t is below the forward rate.

Question (d)

If the yield curve does not move then $r(t, T) = r(0, T - t)$. Remark that

$$f(t, T) = r(0, T) + t \frac{r(0, T) - r(0, t)}{T - t}$$

if the yield curve is upward sloping then

$$\begin{aligned} r(0, T) &\geq r(0, T - t) \quad (\text{because } T > T - t) \\ r(0, T) - r(0, t) &\geq 0 \end{aligned}$$

Hence, an upward sloping curve implies that

$$f(t, T) \geq r(0, T - t)$$

So, when the yield curve does not move the realized interest rates is below the forward rate and the strategy is profitable. **Question (e)**

	$T - t = 1$	$T - t = 2$	$T - t = 3$	$T - t = 4$
$t = 0$	3%	3.3%	3.6%	3.7%
$t = 1$	3.6%	3.9%	3.93%	NA
$t = 2$	4.2%	4.1%	NA	NA
$t = 3$	4.0%	NA	NA	NA

To obtain N euros today, you need $\frac{N}{Z(0)}$ yen. To obtain those yen you can borrow them until date T . You will need to refund $\frac{N}{Z(0)} \frac{1}{P^{yen}(0, T)}$ in yen which is equivalent to pay $N \frac{Z(T)}{Z(0)} \frac{1}{P^{yen}(0, T)}$. As a result, the NPV equals

$$\begin{aligned} NPV_{CB} &= N \left(1 - \frac{Z(T)}{Z(0)} \frac{P^{eur}(0, T)}{P^{yen}(0, T)} \right) \\ &= N \left(1 - \frac{Z(T)}{F(0, T)} \right) \end{aligned}$$

where $F(0, T) := Z(0) \frac{P^{yen}(0, T)}{P^{eur}(0, T)} = Z(0) e^{T(r^{eur}(T) - r^{yen}(T))}$ is the forward exchange rate. So the NPV is positive if and only if

$$Z(T) < F(0, T)$$

i.e. if the future currency rate is lower than the forward currency rate.

In our example, $P^{yen}(0, T) = 1$. If then yen follows the trend indicated by the graph, not only $Z(4)$ might not increase but it could very well decrease generating a gain.

If the yen depreciates 30% in the coming 4 years then $Z(4) = 70\%Z(0)$ and $NPV_{CB} > 30\%$...

2.5 Benefiting from the Abenomics

Exercise 20

In 2012 and 2013, the Japanese government led by Shinzo Abe adopted massive stimulus plans. The move was part of a series of fiscal stimulus decisions sometimes referred to as Abenomics.

As a result, in 2013, the Nikkei 225 appreciated approx. +57% in 2013. On the contrary, the yen depreciated approx. -22% against USD moving from 87 JPY/USD to 105 JPY/USD. In 2013, interest rates were approx. equals zero both in JPY and in USD.

How could a US investor benefit from those Abenomics ?

Consider the two following strategies:

- Strategy 1:
 - Jan, 1st 2013: borrow USD 1M, spot it in JPY and buy a Nikkei 225 ETF
 - Dec 31st 2013: sell the Nikkei ETF, spot the proceeds in USD and redeem the USD loan
- Strategy 2:
 - Jan, 1st 2013: borrow JPY 87M, buy a Nikkei 225 ETF
 - Dec 31st 2013: sell the Nikkei ETF, redeem the JPY loan and spot the proceeds in USD

(a) Compute the final USD value of both strategies. Discuss

(b) Consider the two following strategies:

- Strategy 3:
 - Jan, 1st 2013: “buy” a 1Y future contract on Nikkei 225 with notional JPY 87M.

- Dec, 31st 2013: receive the future payment at expiry and spot it in USD
 - Strategy 4:
 - Jan, 1st 2013: “buy” a 1Y CME future contract on USD/JPY with notional USD 1M.
 - Dec, 31st 2013: receive the future payment at expiry
- (c) Compute strategies 3 et 4 P&Ls. Can you combine them ?
- (d) Eventually, what would have been an appropriate Abenomics “trade” ?

Solution 20

To be written