UNIVERSITÉ PARIS 1
PANTHÉON SORBONNE

Final Exam
Financial Products and Introduction to Pricing

MMMEF & IRFA
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Max.duration: 2 hours 30 min

Instructions: The use of cell phones, tablets, and computers is strictly prohibited. Communication or signalling with others during the exam is strictly prohibited. This might cost you at least 5 points as well as a notification to the Administration who will advise.

Foreword: Write your answers clearly and neatly, as only legible work will receive full credit. On top of that, Bonus points may be awarded for presentation quality. Having said that, I wish you all the best.

With Solutions

Date: October 24, 2025

1 Forward Contracts

Exercise 1.1

Consider a one-year forward contract on one ounce of gold. Assume that it costs €2 per ounce per year to store gold. Assume that the spot price today is €350 and the term structure of interest rates is flat at 10%. Write the formula of the forward price, and provide an approximate numerical value, when the payment of the storage cost is made:

- At the end of the year.
- At the beginning of the year.
- At the middle of the year.

Solution 1.1

The cost of storage increases the cost of carrying the asset and hence increases the forward price.

1. $\frac{350}{ZC} + 2$
2. $\frac{350+2}{ZC}$
3. $\frac{350+2e^{-5\%}}{ZC}$

Exercise 1.2

Today is January 1, 2016 and interest rates are equal to zero. Last year, in 2015, Netflix posted a performance of approximately +134%. That same year, Vale S.A., a Brasilian company specialized in iron extraction lost nearly half of its value. Netflix stock price today is equal to \$117 and Vale S.A. stock price is equal to \$4.3. Neither company announced any dividend payment between January and June.

1. What is the 3 month forward price for each company?
2. Netflix 3 month forward price is proposed at 115. Can you take advantage of this opportunity? If yes, describe the corresponding operations.
3. It turns out that Netflix has announced a dividend payment of \$3 in March but you did not notice it. Is your strategy still valid. What should you do now ?

Solution 1.2

1. Equal to their spot price
2. Since the forward price is too low, you can go long the forward, sell the stock and put the money at the bank. At maturity you will receive: $S_T - S_T + 117 - 115 = 2$.
3. Now the forward price is too high so you should sell the forward, buy the stock by borrowing money. At maturity you will receive: $S_T - S_T - 117 + 115 + 3 = 1$

Exercise 1.3

Consider an asset which price at date t is denoted $S(t)$. Let $P(t,T)$ the ZC price of maturity T. The forward price of maturity T is denoted $F(t,T) \equiv \frac{S(t)}{P(t,T)}$. The log return between two dates t_1 and t_2 of any generic asset with price $X(t)$ is defined as $\mu_X(t_1,t_2) \equiv ln\frac{X(t_2)}{X(t_1)}$.

1. Derive the relationship between the log returns of the Forward, the spot and the ZC.
2. Discuss the previous result. How does this relate to the return of the cash and carry strategy ?
3. What is the return of the forward if the ZC price does not change between t_1 and t_2

Solution 1.3

1. $$ln\left(\frac{S(t_2)}{P(t_2,T)}/\frac{S(t_1)}{P(t_1,T)}\right) = ln\left(\frac{S(t_2)}{S(t_1,T)}\right) - ln(\left(\frac{P(t_2,T)}{P(t_1,T)}\right) = \mu_S - \mu_P$$
2. Being long the forward is equivalent to receive the performance of the spot and pay the cost of borrowing.
3. $\mu_F = \mu_S$

2 Options

Consider a European option with payoff function f, on an underlying with final -random- value S_T. Its price can be computed as $e^{-rT}E(f(S_T))$, where the expectation is calculated under the Risk Neutral Probability. Accordingly, the intrinsic value can be defined as $e^{-rT}f(E(S_T))$, again under the Risk Neutral Probability.

Exercise 2.1

The 1Y discounting factor is equal to 95%. We consider a one year call option with strike price 200. Denote S_0 the current value of the underlying stock price, C_0 the price of the European call and P_0 the price of the European put, each of them with strike 200. No dividend will be paid.

1. Denote IC_0 and IP_0 respectively the intrinsic value of the call, and the intrinsic value of the put. Calculate IC_0 and IP_0 as functions of S_0.
2. Prove that $\forall S_0$, $C_0 \geq IC_0$ and that $P_0 \geq IP_0$.
3. Which name is usually given to the difference between the price of the option and its intrinsic value ?
4. For a generic payoff f, provide a second order approximation of the difference between its price and its intrinsic value, as a function of σ_T^2 the variance of S_T.

5. You are given the possibility to exercise your options before maturity. Denote EC_0 and EP_0 the money you can obtain by exercising right now your option. Write EC_0 and EP_0 as functions of S_0.

   In what follows, we want to understand if it might be interesting to early exercise the option.

6. Calculate $IC_0 - EC_0$. Can you deduce an ordering relationship between C_0 and EC_0 ? Are there situations where you should exercise the call option before maturity ?

7. Same questions for the put option.

Solution 2.1

1. the time value
2. $IC_0 = (S_0 - 190)^+$ and $IP_0 = (190 - S_0)^+$
3. $EC_0 = (S_0 - 200)^+$ and $IP_0 = (200 - S_0)^+$
4. Let $\Delta C_0 = IC_0 - EC_0$. If $S_0 \geq 200$, $\Delta C_0 = 10$, if $S_0 \leq 190$, $\Delta C_0 = 0$ and if $190 \leq S_0 \leq 200$, $\Delta C_0 = S_0 - 190$.
5. Let $\Delta P_0 = IP_0 - EP_0$. If $S_0 \geq 200$, $\Delta P_0 = 0$, if $S_0 \leq 190$, $\Delta P_0 = -10$ and if $190 \leq S_0 \leq 200$, $\Delta C_0 = 200 - S_0$.
6. $\Delta C_0 = IC_0 - EC_0$ is always positive so it is always more interesting to sell the call than to exercise it.
7. $\Delta C_0 = IC_0 - EC_0$ is always positive so necessarily $C > EC$: it is always more interesting to sell the call than to exercise it.
8. $\Delta P_0 = IP_0 - EP_0$ is sometimes positive but becomes negative when S_0 is sufficiently small: so it might be positive to exercise the put option for some values of S_0.

Exercise 2.2

We now take the same assumptions as in the previous exercise but a dividend equal to 15 euros will be paid one day before maturity.

1. What is the relationship between the payoffs of the call option, the put option, S_T and the strike $K = 200$?
2. From that equality, deduce a relationship between C_0, P_0 and S_0
3. Calculate IC_0 then calculate $IC_0 - EC_0$.
4. Are there situations where it might be interesting to exercise the call option before maturity ?

Solution 2.2

1. At maturity $(S_T - K)^+ + (K - S_T)^+ = S_T - K$

2. Taking the expectation of all terms under the RPN yields $\mathbb{E}((S_T - K)^+) + \mathbb{E}()(K - S_T)^+) = ()S_0/ZC - D) - K$. Now taking the discounting value of the previous yields $C_0 - P_0 = (S_0 - 15 \times 95\%) - 190$ hence $C_0 - P_0 \approx S_0 - 205$.

3. $IC \approx (S_0 - 205)^+$.)

4. With dividends, we no longer always have $IC_0 \geq EC_0$. This happens indeed when $S_0 \geq 200$. As a result, when the call In The Money, there might be a value of S_0 where exercising becomes more interesting than selling.

Exercise 2.3

Today, with 1 US dollar you can buy $X_0 = 150$ Japanese Yen (JPY). X is referred to as the USDJPY rate. You plan to travel to Japan in six month time and are ready to spend a budget of USD 10000. The Japanese 6M rate is equal to 1% and the USD 6M rate is equal to 5%. The B&S formula for the call price with continuous dividend rate q is denoted $Call(S_0, K, r, q, \sigma, T)$ and $Put(...)$ for the put.

1. Write the formula of the 6M forward USDJPY rate and provide an approximate numerical value.

2. You expect that the realized USDJPY rate in 6M will be equal to its spot value. What should you do ? Change now, change in 6M or enter into a forward contract ?

3. You want to protect yourself against a 10% or more appreciation of the yen against USD. In that scenario, would X increase or decrease ? Should you buy a call or a put ?

4. To calculate the price of the option you want to buy, which parameters S_0, K, r, q, T should you input in your BS formula ? Explain why.

Solution 2.3

Quotation can either be 1 $USD = X_0$ JPY or 1 $JPY = Y_0$ USD with obviously $X_0 = \frac{1}{Y_0}$. The appreciation of the yen means that we can have less yen with 1 USD hence $X \searrow$ and $Y \nearrow$. The forward value of the Yen compensates the attractiveness of the USD and hence the forward has to be lower than the spot.

1. $F_T = X_0 \times e^{(r_{jpy} - r_{usd})T}$

2. Forward markets predict an appreciation of the yen (i.e. $X_T \leq X_0$). You should not buy a forward. You can either buy yen today and put them on a cash account or, if you are really confident about your prediction, put your dollars in a USD account to benefit from the higher USD rate and change your yens in 6M time.

3. You should buy a put

4. You should use $S_0 = F_T,\ K = 136,\ r = 5\%,\ q = 5\%,\ T = 0.5$. With that set of parameters the forward value is equal to the spot value and the discounting is made at the appropriate (local) rate.